Aims:

Children should be able to use an efficient method, mental or written appropriate to the given task, with understanding. By the end of year 6, children will have been taught, and be secure with, a compact standard method for each operation.

To develop efficient written calculation strategies children need:

- * Secure mental methods which are developed from early years
- * A solid understanding of the number system
- * Practical hands on experience including a range of manipulatives
- * Visual models and images including number lines and arrays
- * Experience of expanded methods to develop understanding and avoid rote learning
- * Secure understanding of each stage before moving onto the next.

Before carrying out a calculation, children will be encouraged to consider:

- * Can I do it in my head? (using rounding, adjustment)
- * The size of an approximate answer (estimation)
- * Could I use jottings to keep track of the calculation?
- * Do I need to use an expanded or compact written method?

Pre requisite skills for written calculations

Addition and subtraction:

- * Do they know all the addition and subtraction facts for all numbers to 20?
- * Do they understand place value and can they partition and then re-partition numbers?
- * Can they add three single digit numbers mentally?
- * Can they add and subtract any pair of two digit numbers mentally?
- * Can they explain their mental strategies orally and record them using informal jottings?

Multiplication and Division:

- * Do they know the 2, 5 and 10 times tables and corresponding division facts?
- * Do they know the result of multiplying by 1 and 0?
- * Do they understand 0 as a place holder?
- * Can they multiply two and three digit numbers by 10 and 100?
- * Can they double and halve two digit numbers mentally?

* Can they use multiplication and division facts they know to derive mentally other multiplication and division facts that they do not know?

* Can they explain their mental strategies orally and record them using informal jottings?

These lists are not exhaustive but are a guide for the teacher as they structure the move from informal to formal methods of calculation. It is vitally important that children's mental methods of calculation are continued to be practised and secured alongside their learning and use of an efficient written method for each operation.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping.	Column method- regrouping. (up to 3 digits)	Column method- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- regrouping. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

Progression of learning in addition, subtraction, multiplication and division across KS1 and KS2

A pathway to teaching calculation methods:

Expanded methods should be viewed as steps towards a standard method and not as methods in themselves.

Before beginning to record in a more refined written format children must have had significant practical work reinforced with appropriate manipulative, models and images.

Teachers will guide pupils to refine their written methods of recording by modelling and asking questions such as "What is the same? What's different?"

Learning will be planned to ensure pupils are encouraged to use and apply what they have learnt to problem solving tasks.

As children move along the pathway it is vital that they practice, reinforce, consolidate, use and apply it to mathematical learning and NOT simply move onto the next step.

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	3 yart yhole 2 3 Balls 2 Bals	4 + 3 = 7 5 3 Use the part-part 10= 6 + 4 whole diagram as shown above to move into the abstract.
Starting at the big- ger number and counting on	Start with the larger number on the bead string and then count on to the smaller num- ber 1 by 1 to find the answer.	12 + 5 = 17 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. This is an essential skill for column addition later.	6 + 5 = 11 Start with the bigger number and use the smaller number to make 10. Use ten frames.	Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 9 + 5 = 14	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	2 more than 5 is 7 Bead strings used for number bonds for ALL numbers to 20	$\begin{array}{c c} & & & \\ \hline \\ & & & \\ \hline \\ \hline$	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

Objective &	Concrete	Pictorial	Abstract
Strategy			
Adding multiples of ten	50= 30 = 20	3 tens + 5 tens = tens 30 + 50 = Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 $40 + \Box = 60$
Use known number facts Part part whole	Children explore ways of making numbers within 20	20 - = = = = = = = = = = = = = = = = = =	+ 1 = 16 $16 - 1 = 1 + = 16 16 - = 1$
Using known facts - understanding flexibility of numbers	$\Box = \Box =$	$\begin{array}{cccc} \vdots & + & \vdots & = & \vdots \\ & & & & & \\ & & & & & \\ & & & & &$	3 + 4 = 7 leads to 30 + 40 = 70 leads to 300 + 400 = 700
Bar model		7 + 3 = 10	23 25 ? 23 + 25 = 48

Objective &	Concrete	Pictorial	Abstract	
Strategy				V'
Add a two digit number and ones	17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	17 + 5 = 22 Use part part whole and number line to model. $17 + 5 = 22$ $16 + 7$ $16 + 7$ $16 + 7$	41 + 8 Explore related facts $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$ $21 + 8 = 9$ $40 + 9 = 49$ $40 + 9 = 49$ $5 + 17 = 22$	
Add a 2 digit num- ber and tens	25 + 10 = 35 Explore that the ones digit does not change	27 + 30 +10 +10 +10 27 37 47 57	27 + 10 = 37 27 + 20 = 47 $27 + \Box = 57$	
Add two 2-digit numbers	Model using dienes , place value counters and numicon	+20 +5 Or +20 +3 +2 47 67 72 47 67 70 $72Use number line and bridge ten using partwhole if necessary.$	25 + 47 $20 + 5$ $40 + 7$ $20 + 40 = 60$ $5 + 7 = 12$ $60 + 12 = 72$	
Add three 1-digit numbers	Combine to make 10 first if possible, or bridge 10 then add third digit	Regroup and draw representation. + $+$ $+$ $+$ $+$ $+$ $+$ $+$ $+$ $+$	4 + 7 + 6 = 10 + 7 $= 17$ Combine the two numbers that make/bridge ten then add on the third.	



Objective &	Concrete	Pictorial	Abstract	
Strategy Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.		Begin with expanded method as in Y3 7836	Y4 •6
	Hundreds Tens One Image: Imag	7 1 5 1 7 1 5 1 • • • • Draw representations using pv grid. Use digit cards using PV grid	+ <u>1351</u> <u>9187</u> <u>1</u> Continue from previous work to carry hundreds as well as tens. Relate to money and measures.	
Y5—add numbers with more than 4 digits. Add decimals with 2 dec- imal places, including money.	As year 4 tens ones tenths hundred tens ones tenths hundred Introduce decimal place value counters and model exchange for addition.	2.37 + 81.79 ths tens ones tents hundred to 00 000 0000 00000 00000 0000 00000 0000 000000 0000 0000000 0000 00000000	72.8 +54.6 127.4 1 1 $f \ge 23 \cdot 59$ $+ f \ge 7 \cdot 55$ $f \ge 3 \cdot 4$	
Y6—add several num- bers of increasing com- plexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away. 6-4=2 4-2=2	$\begin{array}{c} & & & & & & \\ & & & & & & \\ & & & & & $	Written calculations using variations in layout: 7-4 = 3 $\Box = 7 - 4$ 16-9 = 7
Counting back	Move objects away from the group, counting backwards. Move the beads along the bead string as you count backward	Count back in ones using a number line. Start at the bigger number and count back the smaller number showing jumps on the number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts T 'Seven is 3 more than four' 'I am 2 years older than my sister' 5 Pencils 3 Erasers 2 Lay objects to represent bar model.	Count on using a number line to find the difference. $*^{6}$ $+^{6}$ 0 1 2 3 4 5 6 7 8 9 10 11 12	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister?

Objective &	Concrete	Pictorial	Abstract
Strategy			
Represent and use number bonds and related subtraction facts within 20	Link to addition. Use PPW model to model the inverse.		Move to using numbers within the part whole model.
model	If 10 is the whole and 6 is one of the parts, what s the other part? 10-6 = 4	Use pictorial representations to show the part.	7
Make 10	14—5 Make 14 on the numicon (or ten frame). Take 4 away to make ten, then take one more away (cover one up) so that you have taken 5.	Children to represent numicon pictorially. Can they represent this on a number line?	$\begin{array}{c} 14-5\\ \text{Children to solve mentally and explain: how many do we take off first to get to 10? How many left to take off?\\ \text{Children need to know related facts that will help them}\\ \hline 14-5=9\\ 4\\ 1\\ 1\\ 5 \text{ is made up of 4 and 1 so I can subtract 4 to make 10 and then to get to 9} \end{array}$
Bar model			8 2
	5—2 = 3		10 = 8 + 2
			10 = 2 + 8
			10—2 = 8
			10—8 = 2

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	20 - 4 =	20—4 = 16
Partitioning to sub- tract without re- grouping. 'Friendly numbers'	34–13 = 21	Children draw representations of Dienes and cross off. $\begin{array}{c} & & & \\ &$	43—21 = 22
Make ten strategy Progression should be crossing one ten, crossing more than one ten, cross- ing the hundreds.	34-28 Use a bead bar or bead strings to model counting to next ten and the rest.	$\begin{array}{c} & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & &$	93—76 = 17

Objective &	Concrete	Pictorial	Abstract	
Strategy				YX
Column subtraction without regrouping (friendly numbers)	47—32	Calculations 54 -22 32	47 - 24 = 23 $40 and 7$ $20 and 4$ $20 and 3$	
	Use base 10 or Numicon to model making the bigger number and then taking the smaller number away	Draw base 10 representations next to the written number sentence to support under- standing of working out.	needed to lead to clear subtraction understanding.	
Column subtraction with regrouping	Column method (using base 10 and having to exchange) 45-26 1) Start by partitioning 45 2) Exchange one ten for ten more	45 -29 Tens lones 16 HIL DE	$836 - 254 = 582$ $\frac{360}{130} + \frac{7}{130} + \frac{6}{6}$ $= 200 - 50 + \frac{7}{500} + \frac{7}{200} + \frac{6}{200} + \frac{7}{200} + \frac{7}{20} + \frac{7}{200} $	
	3) Subtract the ones, then the tens.	$\begin{array}{c} 30 = 16 \\ 30 = 16 \\ 10 + 6 = 16 \end{array}$ Children may draw base ten or PV counters and cross off.	$728-582=146$ Then move to $477'_2$ $77'_2$ $577'_2$ $77'_2$ $577'_2$ $77'_2$ $57'_2$ $77'_2$ <	R

Objective &	Concrete	Pictorial	Abstract	VIC
Subtracting tens and ones Year 4 subtract with up to 4 digits. Introduce decimal subtrac- tion through context of money	Column method (using base 10 and having to exchange) 45-26 1) Start by partitioning 45 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens. Model process of exchange using Numi- con, base ten and then move to PV coun- ters.	Children to use digit cards and pv counters and show their exchange—see Y3 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.	2 X 5 4 - 1 5 6 2 1 1 9 2 Use the phrase 'exchange' for H/T/U	
Year 5- Subtract with at least 4 dig- its, including money and measures. Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal	As Year 4	Children to draw pv counters and show their exchange—see Y3	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	I RAC
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	

Objective &	Concrete	Pictorial	Abstract	
Strategy				
Doubling	Use practical activities using manip- ultives including cubes and Numicon to demonstrate doubling	Draw pictures to show how to double numbers	Mental recall of doubles. Can they use knowledge of doubles to solve simple word problems?] • •
	$\begin{array}{c} + & \bigcirc & = \\ & & & & \\ & & & \\ & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & &$	Double 4 is 8		
Counting in multi- ples	Count the groups as children are skip counting, children may use their fin- gers as they are skip counting.	Children make representations to show counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of num- bers. 2, 4, 6, 8, 10	PFG
Making equal	The state of the state	2 4 6 8 10 12 14 16 18 20	5, 10, 15, 20, 25 , 3 2 x 4 = 8	
groups and counting the total	Use manipulatives to create equal groups.	Draw CX to show 2 x 3 = 6 Draw and make representations		

Objective &	Concrete	Pictorial	Abstract	V
Strategy				Υ
Repeated addition	Use different objects to add equal groups	Use pictorial including number lines to solve protemfiere are 3 sweets in one bag. How many sweets are in 5 bags altogether?	Write addition sentences to describe objects and pictures. $\underbrace{\begin{array}{c} \hline \\ \hline $	
Understanding ar- rays	Use objects laid out in arrays to find the an- swers to 2 lots 5, 3 lots of 2 etc.	Draw representations of arrays to show under- standing	3 x 2 = 6 2 x 5 = 10	

Objective &	Concrete	Pictorial	Abstract	V)
Strategy				
Doubling	Model doubling using dienes and PV counters.	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.	
	40 + 12 = 52		$ \begin{array}{c} 16 \\ 10 \\ 1 \\ x^2 \\ 20 \\ + 12 \\ = 32 \end{array} $	
Counting in multi-	Count the groups as children are skip	Number lines, counting sticks and bar	Count in multiples of a number aloud.	
ples of 2, 3, 4, 5, 10	counting, children may use their fin-	models should be used to show repre-		
from 0	gers as they are skip counting. Use bar	sentation of counting in multiples.	Write sequences with multiples of	
(repeated addition)	models.	marin marin marin	numbers.	
		YA WAYA YA WA	0, 2, 4, 6, 8, 10	
	5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40	0 5 10 15 20 25 30	0, 3, 6, 9, 12, 15	
		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0, 5, 10, 15, 20, 25 , 30	
	III III IIII ?	3 3 3 3 ?	Missing number $4 \times 12 = 12$ $4 \times 3 =$	

Objective &	Concrete	Pictorial	Abstract	VJ
Strategy				
Multiplication is commutative	Create arrays using counters and cubes and Numicon.	Use representations of arrays to show different calculations and explore commutativity.	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition. 00000 $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$	
Using the Inverse This should be taught alongside division, so pupils learn how they work alongside each other.		$\begin{vmatrix} 4 & 2 \\ \hline 4 & 2 \\ \hline \times & = \\ \hline \times & = \\ \hline \times & = \\ \hline \div & = \\ \hline \div & = \\ \hline \div & = \\ \end{vmatrix}$	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.	SATION X





Objective &	Concrete	Pictorial	Abstract	
Strategy				13.0
Column Multiplication for 3 and 4 digits x 1 digit.	Children can continue to be supported by place value counters at the stage of multiplication.	× 300 20 7 4 1200 80 28 Arrays for identifying factors of numbers	→ 327 × 4 1308	
Column multiplication	Manipulatives may still be used with the cor- responding long multiplication modelled alongside.	Continue to use bar modelling to support prob- lem solving	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	

Objective &	Concrete	Pictorial	Abstract
Strategy			
Multiplying decimals up to 2 decimal plac- es by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.
			3 · 1 9 × 8 2 5 · 5 2



Objective &	Concrete	Pictorial	Abstract	VO
Strategy				
Division as sharing	I have 10 cubes, can you share them equally in 2 groups? Use a range of manipulates	Children use pictures or shapes to share quanti- ties. Move on to sorting circles $\begin{array}{c} & & & & \\ & & & & \\ & & & & \\ & & & & $	12 ÷ 3 = 4	
				
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use number lines for grouping	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?	
	0 5 10 15 20 25 30 35	20 ÷ 5 = ? 5 x ? = 20		

Objective &	Concrete	Pictorial	Abstract	VÍ
Strategy				Y
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.	Continue to use bar modelling to aid solving division problems.	How many groups of 6 in 24?	
		20	24 ÷ 6 = 4	C
	24 divided into groups of 6 = 4	20 ÷ 5 = ? 5 x ? = 20		
	96 ÷ 3 = 32			
Division with arrays		Draw an array and use lines to split the array into groups to make multiplication and division sentences	Find the inverse of multiplication and division sentences by creating eight linking number sentences.	
		$\bigcirc \bigcirc $	7 x 4 = 28	
	Link division to multiplication by creating an array and thinking about the number sentenc-	$\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$	28 ÷ 7 = 4	
	es that can be created.	$\bigcirc \bigcirc $	28 ÷ 4 = 7	
			28 = 7 x 4	
	Eg 15 ÷ 3 = 5 5 x 3 = 15		28 = 4 x 7	
	15 ÷ 5 = 3 3 x 5 = 15		4 = 28 ÷ 7	
			7 = 28 ÷ 4	



Objective &	Concrete	Pictorial	Abstract	VI
Strategy				14
Y4 Divide at least 3 digit numbers by 1 digit. Short Division	Practice chunking using counters or cubes to demonstrate taking 'chunks' away from the starting number	Represent on a number how the 'chunks of groups' are taken away rather than one group at a time division by chunking on. a number line $96 \div 4 = 24$ 10 ± 4 40 80 $96How many lots of 4 attogether?10 \pm 10 \pm 4 = 24$	12 + 3 12 Subtract 3: -3 (1×3) 9 - 3 = 6 6 3 have been Subtract 3: -3 (1×3) 9 - 3 = 6 6 3 have been Subtract 3: -3 (1×3) We reached an answer of 0, so there is no remainder and the answer to 12 + 3 is 4. (1×3)	





Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o <mark>2</mark> 2) <mark>5</mark> 8	t o 2 2) <mark>5</mark> 8 - 4 1	t ∘ 2 9 2) 5 8 <u>- 4 ↓</u> 1 8
Two goes into 5 two times, or 5 tens + 2 = 2 whole tens but there is a remainder!	To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o	t o	t o
2 9 2) 5 8	29	2)58
-4	<u>-4</u>	$\frac{-4}{1}$
18	- <u>18</u>	<u>- 1 8</u>
	<mark>0</mark>	0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.



